Journal of Analysis and Applications
Vol. 20 (2022), No.1, pp.69-80
ISSN: 0972-5954
© SAS International Publications
URL : www.sasip.net

Direct sum of star matrices

K.U. Divya* and K. Somasundaram

Abstract. Let S_n be the symmetric group of order n. The permanent of an $n \times n$ matrix $A = (a_{ij})$ is defined as $\sum_{\sigma \in S_n} \prod_{i=1}^{n} a_{i\sigma(i)}$. Let Ω_n denote the set of all $n \times n$ doubly stochastic matrices. A matrix $B \in \Omega_n$ is said to be a star matrix if $per(\alpha B + (1-\alpha)A \leq \alpha perB + (1-\alpha)perA$, for all $A \in \Omega_n$ and all $\alpha \in [0,1]$. Karuppanchetty and Maria Arulraj [3] proposed the following two conjectures:

(i) The direct sum of two star matrices is a star (also known as the star conjecture).

(ii) The only stars in Ω_n are the direct sum of 2×2 star matrices and identity matrices upto permutations of rows and columns.

In this paper, we derive some sufficient conditions for the direct sum of matrices in Ω_2 to satisfy the inequality of the star conjecture. We also provide some classes of matrices in Ω_n which satisfy the star condition.

AMS Subject Classification (2020): 15A15

Keywords: Doubly stochastic matrices, permanents, star matrices

---

*Corresponding author